

Midterm Exam Solutions

2105623 Optimization of Chemical Processes

12 October 2026 | Version A | 30 questions

Each item includes the original question and all four options, followed by the correct answer and an explanation. Option letters match the Version A examination paper.

Answer key

Question	Answer	Question	Answer	Question	Answer
1	D	11	D	21	D
2	B	12	D	22	A
3	A	13	C	23	C
4	B	14	A	24	A
5	D	15	B	25	A
6	D	16	C	26	D
7	A	17	B	27	C
8	A	18	C	28	B
9	B	19	B	29	B
10	C	20	C	30	A

Reading the solutions

Check the stated assumptions before writing equations. Pay particular attention to mass fractions, binary linking constraints, inventory timing, and the difference between degree constraints and a connected TSP tour.

Questions 1-2

1. Variable product quantity

A blend uses nonnegative feed amounts a and b with impurity fractions 0.01 and 0.07. Product amount p is a decision variable with $p \geq 100$ t. There is no mass loss. Which pair exactly represents mass conservation and a maximum product impurity of 0.03?

- A. $a + b = p$; $0.01 a + 0.07 b \leq 3$
- B. $a + b \geq p$; $0.01 a + 0.07 b \leq 0.03 p$
- C. $a + b = p$; $0.01 a + 0.07 b = 0.03 p$
- D. $a + b = p$; $0.01 a + 0.07 b \leq 0.03 p$

Answer D: $a + b = p$; $0.01 a + 0.07 b \leq 0.03 p$

Explanation

The quality limit scales with actual product amount p , not its lower bound of 100. Equality in the quality constraint would exclude cleaner blends. An inequality in the mass balance would permit unaccounted material.

2. Recognizing a linear reformulation

A blending model has continuous nonnegative a and b , $a + b \geq 100$, and binary y . Its cost is $50a + 30b + 200y$. It includes $b \leq 80y$ and $(0.01a + 0.07b)/(a + b) \leq 0.03$; all other constraints are linear. Which statement is correct?

- A. It becomes an MILP only if a and b are restricted to integers
- B. It can be written as an MILP by multiplying the quality inequality by the strictly positive $a + b$
- C. It must remain nonlinear because the quality is written as a ratio
- D. It can be written as an LP without retaining the binary restriction on y

Answer B: It can be written as an MILP by multiplying the quality inequality by the strictly positive $a + b$

Explanation

The denominator is at least 100, so multiplication preserves the inequality and gives $0.01a + 0.07b \leq 0.03(a + b)$, which is linear. The binary y makes the reformulated model an MILP.

Dropping the binary restriction creates an LP relaxation. It does not give an equivalent formulation of the original model.

Questions 3-4

3. Feasible bound

Feeds A and B contain 1 wt% and 7 wt% impurity. Their flows must sum to 120 t/day, and product impurity must not exceed 3 wt%. There is no mass loss or impurity removal. Ignoring other limits, what is the largest possible B flow?

- A. 40 t/day
- B. 20 t/day
- C. 120 t/day
- D. 60 t/day

Answer A: 40 t/day

Explanation

Substitute $A = 120 - B$ into quality: $1.2 + 0.06B \leq 3.6$. Therefore $B \leq 40$ t/day.

4. Diagnosing an incomplete demand model

A plant must deliver at least 80 t/day and has capacity 100 t/day. An LLM model minimizes $20x$ subject only to $0 \leq x \leq 100$. The solver returns $x = 0$. Which diagnosis and correction are both valid?

- A. The model omits demand; add $x \leq 80$
- B. The model omits demand; add $x \geq 80$
- C. The solver failed; rerun the unchanged model
- D. The objective is incorrect; maximize $20x$ instead

Answer B: The model omits demand; add $x \geq 80$

Explanation

The reported solution is optimal for the encoded model. The missing lower demand bound must be added. Maximizing cost may happen to produce enough but changes the business objective and does not encode demand.

Questions 5-6

5. Simultaneous product specifications

A 100 t blend contains only A and B, with no mass loss. A has 50 wt% useful component and 4 wt% impurity; B has 10 wt% useful component and 1 wt% impurity. The blend needs at least 30 wt% useful component and at most 2 wt% impurity. Which conclusion follows?

- A. Every blend with $a \geq 50$ satisfies both specifications
- B. A feasible blend exists at $a = 100/3$ and $b = 200/3$
- C. A feasible blend exists at $a = 50$ and $b = 50$
- D. No blend is feasible: the useful-component requirement needs $a \geq 50$, but impurity requires $a \leq 100/3$

Answer D: No blend is feasible: the useful-component requirement needs $a \geq 50$, but impurity requires $a \leq 100/3$

Explanation

Using $b = 100 - a$, the useful component gives $10 + 0.40a \geq 30$, hence $a \geq 50$. Impurity gives $1 + 0.03a \leq 2$, hence $a \leq 100/3$. These intervals do not overlap.

6. Binary activation

Let x be a nonnegative flow and y a binary activation variable. The model contains $x \leq 60y$. Which statement must be true?

- A. If $y = 1$, then $x = 60$
- B. If $x = 0$, then $y = 0$
- C. If $y = 1$, then $x > 0$
- D. If $y = 0$, then $x = 0$

Answer D: If $y = 0$, then $x = 0$

Explanation

With $y = 0$, nonnegativity and the upper bound force $x = 0$. With $y = 1$, every flow from 0 to 60 is allowed by this constraint.

Questions 7-8

7. Minimum order

An inactive supplier delivers zero. An active supplier must deliver between 12 and 50 t/day. Let x be a continuous nonnegative delivery and y a binary activation decision. Which constraints express the complete rule?

- A. $12 y \leq x \leq 50 y$
- B. $x \leq 50 y$ only
- C. $12 y \leq x \leq 50$
- D. $12 \leq x \leq 50 y$

Answer A: $12 y \leq x \leq 50 y$

Explanation

Both linking bounds depend on activation. The alternatives force activation, omit the active minimum, or permit delivery while inactive.

8. Optional capacity expansion

A unit has 40 t/day of existing capacity. Binary y selects an expansion providing 30 additional t/day for a fixed daily charge of 400 USD. Processing any tonne costs 25 USD. Which pair correctly represents capacity and daily cost for nonnegative throughput x ?

- A. $x \leq 40 + 30y$; minimize $25x + 400y$
- B. $x \leq 40 + 30y$; minimize $25x + 400xy$
- C. $x \leq 40y + 30$; minimize $25x + 400y$
- D. $x \leq 70y$; minimize $25x + 400y$

Answer A: $x \leq 40 + 30y$; minimize $25x + 400y$

Explanation

At $y = 0$, existing capacity must remain 40; at $y = 1$, total capacity is 70. The expansion charge depends on selection, not on throughput. The other capacity expressions incorrectly remove or reduce existing capacity.

Questions 9-10

9. Deriving an activation bound

A and B form exactly 150 t/day of product without loss. Their impurity contents are 2 wt% and 8 wt%, and the product limit is 4 wt%. B also has a supply limit of 90 t/day. For $B \leq M$ with binary activation y , what is the smallest valid M ? A has no supply limit.

- A. 40 t/day
- B. 50 t/day
- C. 150 t/day
- D. 90 t/day

Answer B: 50 t/day

Explanation

Substitution $A = 150 - B$ gives $3 + 0.06B \leq 6$, so $B \leq 50$. This is tighter than 90 and is attainable at $A = 100$, $B = 50$. A bound of 40 would exclude a feasible blend.

10. Shared capacity and minimum loads

Two optional units have nonnegative throughputs x_P and x_Q and binary indicators y_P and y_Q . Their constraints are $30y_P \leq x_P \leq 50y_P$, $30y_Q \leq x_Q \leq 50y_Q$, and $x_P + x_Q \leq 50$. There are no other constraints. Which additional constraint is implied and therefore does not remove any feasible point?

- A. $y_P + y_Q \geq 1$
- B. $y_P + y_Q = 1$
- C. $y_P + y_Q \leq 1$
- D. $y_P = y_Q$

Answer C: $y_P + y_Q \leq 1$

Explanation

If both units were active, total throughput would be at least 60, violating the shared limit of 50. Neither unit need be active, and either unit alone can operate. Thus at most one is implied, but exactly one is not.

Questions 11-12

11. Exact logical equivalence

Binary y_P and y_Q indicate operation of two units. Binary z must equal 1 if and only if both units operate. Which set represents this rule exactly, without relying on the objective?

- A. $z = y_P + y_Q$
- B. $z \leq y_P$; $z \leq y_Q$ only
- C. $z \geq y_P + y_Q - 1$ only
- D. $z \leq y_P$; $z \leq y_Q$; $z \geq y_P + y_Q - 1$

Answer D: $z \leq y_P$; $z \leq y_Q$; $z \geq y_P + y_Q - 1$

Explanation

The two upper bounds force $z = 0$ whenever either unit is off. The lower bound forces $z = 1$ when both are on. Each one-sided alternative permits a wrong value of z ; $z = y_P + y_Q$ excludes the case in which both units operate.

12. Quality-limited economic value

Exactly 100 t/day must be blended from A and optional B, without loss. A costs 80 USD/t and has 1 wt% impurity; B costs 40 USD/t and has 5 wt% impurity. Product impurity is at most 2 wt%. A is unlimited; B is limited to 60 t/day and incurs a daily activation fee F . At what nonnegative F do the best plans with and without B have equal cost?

- A. 4,000 USD/day
- B. 2,400 USD/day
- C. 500 USD/day
- D. 1,000 USD/day

Answer D: 1,000 USD/day

Explanation

Quality gives $1 + 0.04B \leq 2$, hence $B \leq 25$ t/day. Each tonne replacing A saves 40 USD, so the best active blend saves 1,000 before the fee. Without B the cost is 8,000; with B it is 7,000 + F . They tie at $F = 1,000$.

Questions 13-14

13. Solver interpretation

An LLM-generated formulation is solved, and the solver reports an optimal result. Which conclusion is justified by that report?

- A. The solver has corrected any incorrectly written constraints
- B. Checking the units is unnecessary once an optimum is reported
- C. The encoded model has the reported solution status; its match to the stated process still needs checking
- D. All relevant process requirements must have been included

Answer C: The encoded model has the reported solution status; its match to the stated process still needs checking

Explanation

The solver works on the encoded model. A reported optimum does not establish that omitted or mistranslated process requirements are satisfied.

14. An overly restrictive formulation

A process permits any throughput from 0 to 50 t/day when active and zero when inactive. A model instead imposes $x = 50y$, with y binary. A reviewer verifies that every modeled point is physically allowed. Is that enough to establish an exact formulation?

- A. No; the model excludes allowed points such as $x = 20$, $y = 1$
- B. No; the model permits $x > 50$ when $y = 1$
- C. Yes; checking that modeled points are physically allowed proves exact equivalence
- D. Yes; binary y requires an active unit to run at full capacity

Answer A: No; the model excludes allowed points such as $x = 20$, $y = 1$

Explanation

Exact formulation requires both directions: modeled points must be physically allowed, and every allowed point must be representable. Equality to $50y$ excludes partial-load active operation. The correct upper link is $x \leq 50y$ with $x \geq 0$.

Questions 15-16

15. Coupled refinery yields

A fixed-yield refinery produces 0.35 t gasoline and 0.45 t diesel per tonne of crude. It must supply at least 140 t/day gasoline and 225 t/day diesel. Surplus product may be sold, and there are no other restrictions. What is the minimum crude-processing capacity needed?

- A. 365 t/day
- B. 500 t/day
- C. 900 t/day
- D. 400 t/day

Answer B: 500 t/day

Explanation

Gasoline requires $q \geq 140/0.35 = 400$, while diesel requires $q \geq 225/0.45 = 500$. Both products come from the same crude, so the required capacity is the maximum, 500, not the sum. At $q = 500$, gasoline output is 175 and diesel output is 225.

16. Refinery: profit objective

A refinery sells gasoline g and diesel d , both in t/day, at 700 and 600 USD/t. It purchases crude at rate q t/day for 450 USD/t and pays a processing cost of 30 USD per tonne of crude. There are no other costs or revenues. Which objective represents daily profit maximization?

- A. Maximize $700g + 600d - 30(g + d)$
- B. Minimize $700g + 600d - 480q$
- C. Maximize $700g + 600d - 480q$
- D. Maximize $700g + 600d - 450q$

Answer C: Maximize $700g + 600d - 480q$

Explanation

Revenue is $700g + 600d$. Crude purchase and processing together cost $(450 + 30)q = 480q$. Profit is revenue minus these costs, in USD/day.

Questions 17-18

17. Capacity with a campaign changeover

A refinery unit processes daily amounts a and b of two crude grades in separate campaigns, at 40 and 25 t/h. Binary y_A and y_B indicate positive campaigns, enforced elsewhere by valid linking bounds. If both campaigns operate, exactly one 2-hour changeover is required. Binary z is already constrained to equal 1 exactly when both operate. Which constraint represents a 16-hour daily limit?

- A. $a/40 \leq 16$ and $b/25 \leq 16$ only
- B. $a/40 + b/25 + 2z \leq 16$
- C. $a/40 + b/25 + 2(y_A + y_B) \leq 16$
- D. $a/40 + b/25 \leq 16 + 2z$

Answer B: $a/40 + b/25 + 2z \leq 16$

Explanation

Processing time is $a/40 + b/25$. The single changeover consumes two additional hours only when $z = 1$. Charging $2(y_A + y_B)$ charges a single campaign and double-counts the changeover when both run.

18. Refinery: product balance

A simplified refinery processes crude amounts c_1 and c_2 in t/day. Each tonne of crude 1 yields 0.40 t of gasoline, and each tonne of crude 2 yields 0.60 t of gasoline. The remaining mass forms other products. There is no gasoline inventory or loss, and all gasoline is sold at rate g . Which gasoline balance is correct?

- A. $g = c_1 + c_2$
- B. $g \leq 0.40 c_1 + 0.60 c_2$ only
- C. $g = 0.40 c_1 + 0.60 c_2$
- D. $g = 0.60 c_1 + 0.40 c_2$

Answer C: $g = 0.40 c_1 + 0.60 c_2$

Explanation

All gasoline produced must be sold because no gasoline inventory or loss is allowed. The balance therefore equates sales to the sum of the two gasoline-yield contributions.

Questions 19-20

19. Refinery: sulfur limit

A refinery blends exactly 100 t/day of fuel from streams A and B. Their sulfur contents are 0.2 wt% and 1.0 wt%. The fuel may contain at most 0.5 wt% sulfur. There is no sulfur removal or mass loss. For flows a and b in t/day, which sulfur constraint is correct?

- A. $0.002 a + 0.010 b \leq 0.005$
- B. $0.002 a + 0.010 b \leq 0.5$
- C. $0.2 a + 1.0 b \leq 0.5$
- D. $0.002 a + 0.010 b \geq 0.5$

Answer B: $0.002 a + 0.010 b \leq 0.5$

Explanation

The maximum sulfur flow is $0.005(100) = 0.5$ t/day. The feed percentages must also be converted to fractions before multiplying mass flows.

20. Cost with terminal inventory value

Production x_1 and x_2 costs 6 and 9 USD/t. Inventory s_1 carried from period 1 to 2 costs 2 USD/t. Binary y_1 and y_2 incur setup costs of 150 USD per active period. Terminal inventory s_2 has a recoverable value of 3 USD/t after period 2 and no further holding charge. Which objective minimizes net cost?

- A. $6x_1 + 9x_2 - 2s_1 + 150(y_1 + y_2) - 3s_2$
- B. $6x_1 + 9x_2 + 2s_1 + 150(y_1 + y_2) + 3s_2$
- C. $6x_1 + 9x_2 + 2s_1 + 150(y_1 + y_2) - 3s_2$
- D. $6x_1 + 9x_2 + 2(s_1 + s_2) + 150(y_1 + y_2) - 3s_2$

Answer C: $6x_1 + 9x_2 + 2s_1 + 150(y_1 + y_2) - 3s_2$

Explanation

Production, interperiod holding, and setup are costs. The terminal recovery is a credit and is subtracted. Charging holding on s_2 or treating holding on s_1 as a credit contradicts the stated timing.

Questions 21-22

21. Inventory with a timed loss

At the start of period t , 10% of the opening inventory s_{Prev} is lost before production begins. New production x does not suffer this loss. Demand d is then delivered, leaving closing inventory s_{Next} . All quantities are tonnes; there are no other flows. Which balance is correct?

A. $s_{\text{Prev}} + x = 0.90d + s_{\text{Next}}$

B. $s_{\text{Prev}} + x = d + 0.90s_{\text{Next}}$

C. $0.90(s_{\text{Prev}} + x) = d + s_{\text{Next}}$

D. $0.90s_{\text{Prev}} + x = d + s_{\text{Next}}$

Answer D: $0.90s_{\text{Prev}} + x = d + s_{\text{Next}}$

Explanation

Only opening inventory is exposed to the loss. The available amount is therefore $0.90s_{\text{Prev}} + x$. Applying 0.90 to production or to outgoing demand or closing stock changes the stated physical sequence.

22. Inventory required by future capacity

A two-period plant starts with zero stock. Demands are 20 t and 40 t. Production capacities are 50 t and 30 t. Inventory is nonnegative, final stock is zero, and no losses, purchases, or backlog are allowed. What is the minimum inventory that must be carried from period 1 to period 2?

A. 10 t

B. 0 t

C. 20 t

D. 30 t

Answer A: 10 t

Explanation

Period 2 requires $s_1 + x_2 = 40$ and has $x_2 \leq 30$, so $s_1 \geq 10$. It is attainable with $x_1 = 30$ and $x_2 = 30$, meeting both capacities and demands.

Questions 23-24

23. Terminal inventory

A two-period plan starts with 12 t of inventory, must deliver 38 t and 54 t in the two periods, and must finish with exactly 6 t. There are no losses, purchases, or disposal. What total production is required over both periods?

- A. 92 t
- B. 104 t
- C. 86 t
- D. 98 t

Answer C: 86 t

Explanation

Over both periods, $12 + \text{total production} = 38 + 54 + 6$. Total production is therefore 86 t.

24. Aggregate feasibility versus storage capacity

A plant starts and ends a two-period horizon with zero stock. Demands are 30 t and 50 t, and a plan produces 40 t in each period. Production capacity is 50 t per period. Stock may not exceed 5 t at either period end; no losses, disposal, purchases, or backlog are allowed. Which assessment is correct?

- A. The plan violates storage capacity after period 1, despite meeting total demand
- B. The plan violates period-2 production capacity
- C. The plan is feasible because total production and total demand both equal 80
- D. The plan fails period-1 demand by 10 t

Answer A: The plan violates storage capacity after period 1, despite meeting total demand

Explanation

The first balance gives $s_1 = 40 - 30 = 10$, above the 5 t storage limit. The second balance would give $s_2 = 10 + 40 - 50 = 0$. Aggregate balance and production capacity alone therefore miss the storage violation.

Questions 25-26

25. Choosing between feasible production plans

Demands are 40 t and 30 t over two periods. Initial and final inventory are zero; no shortages or losses are allowed. Production costs are 8 and 6 USD/t, interperiod holding costs 2 USD/t, and each period with positive production incurs a 150 USD setup. Production capacity is 70 t per period and storage capacity is 30 t. Which plan has the lowest total cost?

- A. Produce (70, 0) t; activate period 1 only
- B. Produce (60, 10) t; activate both periods
- C. Produce (50, 20) t; activate both periods
- D. Produce (40, 30) t; activate both periods

Answer A: Produce (70, 0) t; activate period 1 only

Explanation

All listed plans are feasible. Plan (70, 0) carries 30 t and costs $560 + 60 + 150 = 770$ USD. Plan (40, 30) costs $320 + 180 + 300 = 800$ USD. Plan (50, 20) costs $400 + 120 + 20 + 300 = 840$ USD. Plan (60, 10) costs $480 + 60 + 40 + 300 = 880$ USD. For plan (70, 0), avoiding the second setup outweighs the higher early production cost and holding cost.

In the printed option order: A = 770, B = 880, C = 840, and D = 800 USD. Thus A saves 30 USD relative to the next-best listed plan.

26. TSP: arc variables

In a directed traveling salesman model, one vehicle must visit each city exactly once and return to its starting city. Travel directly from a city to itself is excluded. For distinct cities i and j , which definition of x_{ij} represents route selection?

- A. x_{ij} is the fixed distance from i to j
- B. x_{ij} is the number assigned to city j
- C. x_{ij} is a continuous nonnegative amount of fuel used in city i
- D. x_{ij} is binary and equals 1 if the route uses the arc from i to j

Answer D: x_{ij} is binary and equals 1 if the route uses the arc from i to j

Explanation

Route selection is a yes/no decision for each directed arc. Travel costs or distances are parameters, not the arc-selection variables.

Questions 27-28

27. TSP: route cost

A directed traveling salesman model has city set V . Binary x_{ij} indicates whether the arc from i to j is used, and c_{ij} is its known travel cost. Arcs with $i = j$ are excluded. Which objective minimizes total route cost?

- A. Minimize sum over i in V and j in $V, j \neq i$, of x_{ij} only
- B. Minimize sum over i in V and j in $V, j \neq i$, of c_{ij} only
- C. Minimize sum over i in V and j in $V, j \neq i$, of $c_{ij} x_{ij}$
- D. Maximize sum over i in V and j in $V, j \neq i$, of $c_{ij} x_{ij}$

Answer C: Minimize sum over i in V and j in $V, j \neq i$, of $c_{ij} x_{ij}$

Explanation

A selected arc contributes its cost, while an unselected arc contributes zero. Summing x_{ij} alone counts arcs and summing c_{ij} alone does not depend on the chosen route.

28. Equivalent connectivity constraints

An incomplete directed five-city TSP model currently imposes only binary arc domains, no self-arcs, and exactly one outgoing arc per city. Incoming-degree constraints have not yet been added. Let $S = \{1, 2\}$, and let $I = x_{12} + x_{21}$. Which constraint is algebraically equivalent to $I \leq 1$ under these outgoing-degree equations?

- A. $x_{13} + x_{14} + x_{15} + x_{23} + x_{24} + x_{25} \leq 1$
- B. $x_{13} + x_{14} + x_{15} + x_{23} + x_{24} + x_{25} \geq 1$
- C. $x_{13} + x_{14} + x_{15} + x_{23} + x_{24} + x_{25} = 1$
- D. $x_{31} + x_{32} + x_{41} + x_{42} + x_{51} + x_{52} \geq 1$

Answer B: $x_{13} + x_{14} + x_{15} + x_{23} + x_{24} + x_{25} \geq 1$

Explanation

The outgoing equations for the two cities give $I + L = 2$, where L counts arcs leaving S . Hence $I \leq 1$ is equivalent to $L \geq 1$. Incoming arcs are not linked by outgoing-degree equations alone. Requiring exactly one leaving arc would exclude tours that leave S twice.

Question 29

29. TSP: disconnected cycles

A directed traveling salesman formulation for five cities requires exactly one incoming and one outgoing arc at each city, but has no subtour-elimination constraints. Which selected arcs show that these degree constraints alone do not guarantee a single tour?

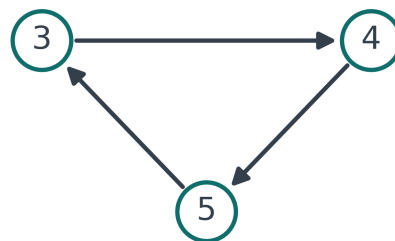
- A. 1 to 2, 1 to 3, 3 to 4, 4 to 5, 5 to 1
- B. 1 to 2, 2 to 1, 3 to 4, 4 to 5, 5 to 3
- C. 1 to 2, 2 to 3, 3 to 4, 4 to 5
- D. 1 to 2, 2 to 3, 3 to 4, 4 to 5, 5 to 1

Answer B: 1 to 2, 2 to 1, 3 to 4, 4 to 5, 5 to 3

Explanation

The arcs 1 to 2 and 2 to 1 form a two-city cycle, while 3 to 4, 4 to 5, and 5 to 3 form a separate three-city cycle. Every city has one arrival and one departure, but there is no single five-city tour. The other options either form a full tour or violate the degree requirements.

One arrival and one departure per city, but two separate cycles



Question 30

30. TSP: a subtour constraint

For a directed traveling salesman model with five cities and no self-arcs, let $S = \{1, 2, 3\}$. Binary x_{ij} selects an arc from i to j . Which valid constraint excludes a three-city cycle visiting all cities in S while preserving every full five-city tour?

- A. $x_{12} + x_{13} + x_{21} + x_{23} + x_{31} + x_{32} \leq 2$
- B. $x_{12} + x_{13} + x_{21} + x_{23} + x_{31} + x_{32} = 3$
- C. $x_{12} + x_{13} + x_{21} + x_{23} + x_{31} + x_{32} \geq 3$
- D. $x_{12} + x_{13} + x_{21} + x_{23} + x_{31} + x_{32} \leq 1$

Answer A: $x_{12} + x_{13} + x_{21} + x_{23} + x_{31} + x_{32} \leq 2$

Explanation

A full five-city tour uses at most $|S| - 1 = 2$ arcs internal to S . A cycle through all three cities in S uses three internal arcs and is excluded. Some full tours use two internal arcs, so a bound of 1 is too restrictive. This constraint alone does not exclude every two-city cycle inside S ; those require constraints for smaller subsets.

