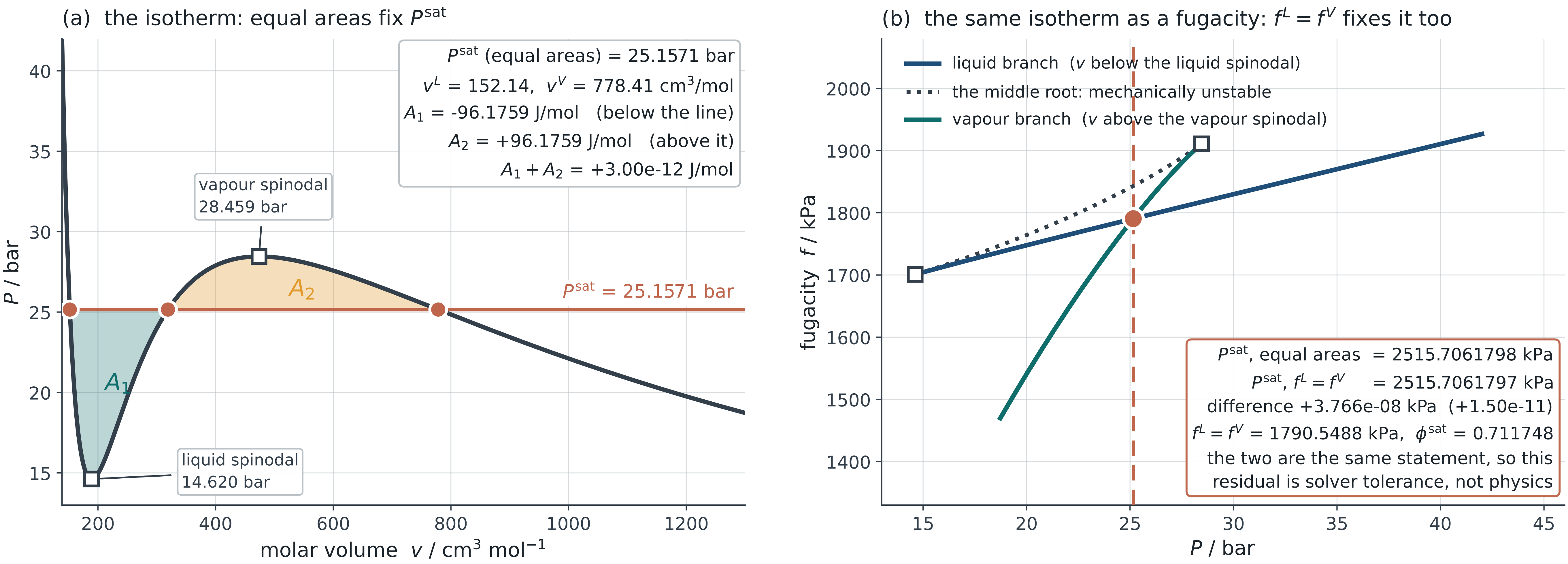
Equal areas and equal fugacities are one statement: $\int_{v^L}^{v^V} v\,dP = g^V - g^L = RT\ln(f^V/f^L) = 0$



n-butane at 400 K (T_r = 0.941), Peng-Robinson with T_c = 425.12 K, P_c = 37.96 bar, ω = 0.2 [Yaws, log10(mmHg), T in C; Tc/Pc/omega Poling et al.], 1976 α.
LEFT: P^{sat} from Brent on $F(v^V) - F(v^L) - P^{sat}(v^V - v^L)$, with F the closed-form antiderivative of $P(v)$;
the two areas are then re-integrated numerically with `scipy.integrate.quad` as an independent check.
RIGHT: $\ln\phi = G^{res}/RT$ from `PengRobinson.gres_RT` at $Z = Pv/RT$ along the same isotherm — that is what puts the unstable middle branch on the figure;
P^{sat} from `vlekit.cubic.psat`. Open squares are the spinodals, where a branch runs out of roots.
ASSUMED: only the constants above and the choice of 400 K, picked because both spinodal pressures are positive there so the construction fits on one linear axis.
Quadrature check: $A_1 + A_2$ = +3.00e-12 J/mol against the closed form's -5.68e-14 J/mol.