Individual assignment · Verification across optimization models

100 raw points · Individual submission

Deadline and course-grade weighting will be announced by the instructor. This assignment is a stand-alone problem set, not a replacement for the computational project.

Submit one PDF report with numbered answers and one runnable notebook or Python script. Include mathematical work, software versions, solver status, objective values and independent feasibility checks. A screenshot of a solver answer alone is insufficient.

You may consult course materials and documentation. Explain your own reasoning, cite external assistance and disclose any AI or code assistance. Do not submit another student's derivations or results. Aim for 5–8 report pages plus code.

1. LP certificate and sensitivity · 25 points

Maximize $4x+3y$ subject to $x+y\le5$, $2x+y\le8$ and $x,y\ge0$.

(5 points) Form the dual using nonnegative resource prices u,v.

(8 points) Find primal and dual optima. Verify feasibility, equality of objective values and every complementary-slackness product.

(6 points) Change only the first RHS to b_1, keeping the objective and second RHS fixed. Derive the complete closed interval on which the intersection of both resource lines is feasible and certified optimal by the original dual prices. State how endpoint prices should be interpreted.

(6 points) Predict and verify the optimal profit at $b_1=5.5$. Explain why extrapolating the same price to $b_1=9$ fails.

2. Integer production and start-up cost · 20 points

Maximize $P=7a+10b-2z$ subject to $2a+3b\le10$, $a\le4z$, a,b nonnegative integers, and $z\in\{0,1\}$. The variables a,b count batches. When enabled, product a has a separate operating limit of4 batches. Only product a requires the start-up decision z, which incurs a fixed cost of2 over this planning period.

(4 points) Explain both implications of $a\le4z$. Does $z=1$ force $a>0$? Explain why $M=4$ is valid for this model.

(5 points) Form and solve the LP relaxation by allowing $a,b\ge0$ to be real and $0\le z\le1$. Give its optimum and a mathematical upper-bound certificate.

(4 points) Before using exhaustive enumeration, verify the candidate $(a,b,z)=(2,2,1)$ and use its value with the LP bound to bracket the integer optimum. Can these bounds alone certify the candidate as optimal?

(7 points) Complete a bounded enumeration for both values of z. For each feasible a, calculate the largest feasible integer b. Use your table to certify the global integer optimum.

3. Unconstrained iterations and Armijo · 20 points

Minimize $f(u,v)=(u-1)^2+2(v+1)^2$ from $r_0=(3,1)$. Use the Euclidean norm for the gradient stopping test. For Armijo, accept equality: $f(r+\alpha p)\le f(r)+c_1\alpha\nabla f(r)^Tp$.

(5 points) Compute gradient and Hessian. Classify the global minimizer.

(5 points) Take one negative-gradient step with fixed $\alpha=0.1$. Report the new point and objective.

(5 points) Restart at r_0 and use $p=-\nabla f$. Apply Armijo with $c_1=0.1$, $\rho=0.5$ and initial $\alpha=1$. Show the candidate point, actual value, target and decision at every trial until first acceptance. Use exact arithmetic for the hand calculation.

(5 points) Continue from the accepted point. Recompute $p=-\nabla f$ and reset α to1 at each outer iteration. Stop when $\|\nabla f\|_2<10^{-8}$. Provide the point, gradient, accepted α, objective and number of tested trials per iteration. Use safeguards of100 outer iterations and40 trials per search; report failure if a safeguard is reached. Compare with fixed $\alpha=0.1$.

4. Constrained NLP and a capacity decision · 35 points

Minimize $F(x,y)=(x-4)^2+2(y-3)^2$ subject to $x+y\le b$, $x,y\ge 0$, and $b\ge 0$. Initially $b=4$. Use $g_1=x+y-b$, $g_2=-x$, $g_3=-y$ and $L=F+\Sigma\lambda_i g_i$. Treat F as a monetary operating loss over one planning period. Extra-capacity charges in part(d) apply to that same period.

(8 points) Derive the candidate with $x,y>0$ and active capacity. Compute all multipliers and F at $b=4$.

(8 points) Verify every KKT condition and explain whether it certifies a global optimum.

(7 points) Derive the complete parameter interval on which the capacity-active formula is feasible and satisfies KKT. Separately identify where $x,y>0$ and where $\lambda_1>0$. Explain what changes at each endpoint.

(7 points) At $b=4$, consider an indivisible offer that increases b to4.5 and charges3.8 per added capacity unit for this same planning period. Compare the exact reduction in optimized operating loss with its first-order prediction. Calculate the offer cost and exact net benefit (loss reduction minus offer cost), then decide whether to accept this fixed offer.

(5 points) At $b=4$, run an NLP solver from (0,0), (1,1) and (4,3); the last start is intentionally infeasible. For each run report success/message, point and objective. Independently report $\max(0,\max_i g_i)$, $\max(0,\max_i(-\lambda_i))$, $\|\nabla F+\Sigma\lambda_i\nabla g_i\|_\infty$ and $\max_i|\lambda_i g_i|$. State how multipliers are obtained and the tolerances used. Diagnose any failed run.

Submission checklist

All four parts are answered. Figures label axes and units. Code runs from top to bottom. Results are compared with analytic calculations. Sensitivity conclusions state the parameter change and valid interval. Attach source code and any data you created.