

MILP · Worked solutions

Model and units

Baseline: maximize $8a+6b-10y_A-8y_B$, subject to $2a+b\leq 12$, $a+b\leq 10$, $a\leq 6y_A$, $b\leq 8y_B$, $a,b\geq 0$ continuous, y_A,y_B binary. Money: \$1,000/week. Reset baseline for each exercise.

Exercise 1. Integer production

Maximize $3x+2y$ with $x+y\leq 4$, $2x+y\leq 6.5$, and nonnegative integer x,y . Find the optimum and verify it by enumeration.

For $y=0,1,2,3,4$ the best allowed x is 3,2,2,1,0 respectively, with values 9,8,10,9,8. Therefore the unique optimum is (2,2), value 10. It uses resources 4 and 6, within limits 4 and 6.5.

Exercise 2. Sales cap 6

Reset the plant model to baseline, then replace $a+b\leq 10$ with $a+b\leq 6$. Compare all four opening patterns.

Neither gives 0. A only: $a=6$, net $48-10=38$. B only: $b=6$, net $36-8=28$. Both: with the tighter sales cap, contribution is maximized by $a=6,b=0$, so net $48-18=30$. Opening B is allowed without producing B but wastes its fixed cost. Best is A only, profit 38.

Exercise 3. At most one line

Use baseline data and add $y_A+y_B\leq 1$. What is the best opening and production plan?

Both is excluded. Neither gives 0, A only gives 38, B only gives 40. Choose $y_A=0,y_B=1,a=0,b=8$. Labor use $8\leq 12$ and sales $8\leq 10$. Profit $48-8=40$.

Exercise 4. A module costing 8

Use baseline plant data plus an optional module adding 4 labor units. Its weekly cost is 8. Compare buy and do not buy.

Without buying, best profit is 46 at $(a,b)=(2,8)$. Buying changes labor capacity to 16. Open both and produce (6,4): labor 16, sales 10, operating profit $48+24-18=54$, then subtract module cost 8 to get 46. Both purchase states are optimal. One returned vector does not establish uniqueness.

Exercise 5. A minimum batch of 4

Reset to baseline and add $a\geq 4y_A$. Compare opening patterns and determine the optimum.

A only still gives 38. B only still gives 40. With both open and $a\geq 4$, the labor limit is binding and $b\leq 12-2a$. Contribution $8a+6b\leq 72-4a$, which decreases with a , so choose $a=4,b=4$. Net profit is $32+24-18=38$. Choose B only with profit 40.

Exercise 6. A remaining optimality gap

A maximization MILP has feasible incumbent value 46 and a valid global upper bound 48. What can be concluded?

The unknown optimum lies in $[46, 48]$. A better feasible plan may exist, but its improvement is at most 2 objective units. This is not yet proof that 46 is exactly optimal. A node upper bound is not a feasible integer value; do not report 48 as an achieved profit.

Exit Ticket 1. Linking

$y_A=0$ gives $a \leq 0$ and $a \geq 0$, so $a=0$. $y_A=1$ gives $3 \leq a \leq 6$. Opening does not force full capacity.

Exit Ticket 2. Relaxation

The relaxed $y_A=1/3$ is not binary. It pays only $10/3$ rather than 10 for opening A. Relaxation enlarges the feasible set, so $158/3 \approx 52.667$ is a valid upper bound, but this plan is inadmissible for the MILP, whose optimum is 46.

Exit Ticket 3. Purchase

Gross operating improvement $= 54 - 46 = 8$. Cost $= 9$. Incremental net benefit $= 8 - 9 = -1$. Buy gives 45; do not buy gives 46. Reject the purchase; best total profit stays 46.

Exit Ticket 4. Pruning

For maximization, node bounds 45 and 46 cannot improve incumbent 46, so prune them when seeking one optimum. Bound 48 may improve it and needs further work. The global certificate is $46 \leq \text{optimum} \leq 48$. A bound of 48 does not prove a feasible integer solution at 48 exists. Keeping all alternative optima would require more care with equality pruning.