

Convexity and unconstrained optimization

Independent worksheet

Show derivatives, substitutions, and the reasoning behind each classification. All functions use dimensionless illustrative coordinates. Questions 5–7 correspond to Practice A–C in the slides. Complete the core examples in class; use the remaining questions for follow-up practice.

1. Convex sets

Classify $C1 = \{x: x_1 \geq 0, x_2 \geq 0, x_1 + x_2 \leq 4\}$ and $C2 = \{x: x_1^2 + x_2^2 \geq 1\}$. Prove convexity or give two feasible endpoints with an infeasible midpoint.

Working: _____

2. Function classification

On $\mathbb{R}$, classify x^4 , $-x^2$, and $q(x) = (x^2 - 1)^2$. State whether a global minimizer exists and whether it is unique.

Working: _____

3. Two gradient descent steps

Let $f(u, v) = (u - 1)^2 + 4(v + 2)^2$. Start at $x_0 = (3, 0)$. Use $\alpha = 0.1$ to compute two gradient descent steps, including the gradient and objective at each iterate.

Working: _____

4. The step-size boundary

For the same quadratic, derive the range of constant positive α that converges from every initial point. Explain $\alpha = 0.25$ and $\alpha = 0.30$ from $x_0 = (3, 0)$.

Working: _____

5. Newton and a coupled quadratic (Practice A)

Minimize $h(x,y)=x^2+2xy+3y^2-4x-8y+7$ on $\mathbb{R}^2$. Derive the gradient, Hessian, eigenvalues, stationary point, and its classification.

Working: _____

6. Nonquadratic Newton (Practice B)

Take one full Newton step for $r(x)=x^4+x^2$ starting at $x_0=2$. Check the new objective. Has the method reached the minimizer?

Working: _____

7. Stationarity and a failed Newton step

For $q(x)=(x^2-1)^2$, calculate a full Newton step from 0.2. Explain why it increases q . Separately explain why x^4 and $-x^4$ cannot be classified from their first two derivatives at zero (Practice C).

Working: _____

8. Lab interpretation

Run the five fixed step sizes and the five nonconvex starts in the notebook. Report the iterate, objective, gradient norm, stopping reason, and your classification. Do not use solver success alone as proof.

Working: _____
